Distributed Cooperative Spectrum Sensing from Sub-Nyquist Samples for Cognitive Radios

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***Abstract*—Distributed collaborative spectrum sensing has been considered for Cognitive Radio (CR) in order to cope with fading and shadowing effects that affect a single CR performance, without the communication overhead of centralized cooperation through a fusion center. In this paper, we consider collaborative spectrum sensing by a distributed network of CRs from sub-Nyquist samples to overcome the sampling rate bottleneck of the wideband signals a CR usually deals with. We present a joint reconstruction algorithm, Randomized Distributed Simultaneous Iterative Hard Thresholding (RDSIHT) that adapts the original IHT to block sparse and matrix (simultaneous) inputs, as well as distributed collaboration settings. An observation vector is passed around the network as a random walk process, and updated at each iteration by one of the CRs. Simulations show that our algorithm outperforms a distributed collaborative scheme based on the One-Step Greedy Algorithm (OSGA) using randomized gossip, and that its performance converges to that of its centralized version.**

I. Introduction

The traditional task of spectrum sensing has been revisited with the emergence of sensor networks [1], [2]. In such settings, a group of receivers senses the surroundings and share some information about the measured spectrum. The sharing can be either soft or hard, namely the sensors share their measurements themselves or only local binary decisions about the presence or absence of a signal. This sharing process can be carried out in two ways: centralized fusion, where the shared data is sent to a fusion center which processes it jointly and transmits its decision back to the sensors, or distributed collaboration where the sensors only communicate with their neighbors.

Collaboration in a sensing network allows to cope with practical issues such as path loss, fading and shadowing [2]. Cooperation has been shown to improve the detection performance and relax sensitivity requirements by exploiting spatial diversity [3], [4]. The authors in [3], [4] quantify the effect of collaboration on the probabilities of detection and false alarm in the Nyquist regime and in centralized settings. In [3], an OR-rule based on the binary decisions of the sensors is used as a fusion rule, whereas a joint optimization problem is solved in [4] to find the optimal decision threshold.

Distributed cooperative spectrum sensing has recently received renewed attention with the development of Cognitive Radio (CR), considered as a promising solution to the ever-increasing spectrum crowdedness [5], [6], [7]. Secondary users would opportunistically access frequency bands left vacant by their primary owners, called white space or spectrum holes, increasing spectral efficiency. Spectrum sensing is an essential task in the CR cycle [7]. Indeed, a CR should be able to constantly monitor the spectrum and detect the primary users (PUs) activity, reliably and fast [8], [9]. To cope with fading and shadowing effects, distributed networks of CRs have been considered in order to avoid the communication overhead of a centralized approach including a fusion center. A distributed approach is considered in [10] to estimate the power distribution in space and frequency. The authors use a discretized grid both in space and frequency and do not recover the emitted signal continuous support. In [11], each CR measures the received energy over a certain sensing time and a consensus algorithm is used to cooperatively detect a single signal only. The authors in [12] consider a single transmission as well and recover its power spectral density. Several decision statistics based on the latter are then derived to decide about the presence or absence of the PU.

In order to increase the chance to find an unoccupied spectral band, the CR has to sense a wide band of spectrum. Nyquist rates of wideband signals are high and can even exceed today's best analog-to-digital converters (ADCs) front-end bandwidths. Moreover, such high sampling rates generate a large number of samples to process, affecting speed and power consumption. Several works thus consider spectrum sensing from sub-Nyquist samples, assuming that the input signal is sparse in the frequency domain. In [13] and [14], the joint support of the sparse signal is recovered from compressive samples acquired by different CRs, using a distributed iterative thresholding algorithm and an approximate message passing approach, respectively. In [15], [16], two schemes are proposed. When the channel state information (CSI) is unknown, only the signal support is recovered, whereas if it is known, the spectrum can be recovered as well. In the first case, the average decision value is computed via an average consensus technique while the spectrum is recovered by combining consensus averaging with sparsity constrained linear regression. However, in all of the above works, in order to derive their reconstruction scheme, the authors exploit a mathematical relation between sub-Nyquist and Nyquist samples, whereas no specific sampling scheme is provided. Moreover, the wideband spectrum is divided into predefined channels, which are each represented by one single entry of the sparse vector to recover. These approaches cannot be naturally extended from a single vector per CR to the block sparse case in distributed settings.

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In this work, we propose a distributed collaborative spectrum sensing method from samples of a multiband signal acquired at a sub-Nyquist rate at each CR. We use the modulated wideband converter (MWC) [17] for the sampling stage. Each CR samples the wideband sparse signal suffering from different effects of fading and shadowing. A single vector, computed from the low rate samples, is passed in the network rather than the samples themselves to reduce communication overhead. We derive a reconstruction algorithm, Randomized Distributed Iterative Hard Thresholding (RDSIHT), that adapts our centralized BSIHT [18] to distributed collaboration. When a CR receives this vector, it performs local computation to update it and then update its estimate of the signal support accordingly. Finally, the vector is sent to a neighbor CR, chosen according to the random walk probability. We do not assume any *a priori* knowledge on the CSI. Simulations show that RDSIHT outperforms a distributed collaborative version of the One-Step Greedy Algorithm (OSGA) presented in [19] using randomized gossip, and that its performance converges to the centralized version, as expected.

This paper is organized as follows. In Section II, we present the models of the transmitted and received signals as well as the CR network. Sections III and IV describe the individual sub-Nyquist sampling process and joint distributed support recovery stage, respectively. Numerical experiments are presented in Section V.

II. Signal and Network Models

A. Transmitted Signal Multiband Model

Let $x(t)$ be a real-valued continuous-time signal, supported on $\mathcal{F} = [-1/2T_{\text{Nyq}}, +1/2T_{\text{Nyq}}]$ and composed of up to N_{sig} transmissions, such that

$$x(t) = \sum_{i=1}^{N_{\text{sig}}} s_i(t), \tag{1}$$

where $s_i(t)$ is a bandpass process. The single-sided bandwidth of each transmission is assumed to not exceed B. Formally, the Fourier transform of $x(t)$ defined by

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \mathrm{d}t \tag{2}$$

is zero for every $f \notin \mathcal{F}$. We denote by $f_{\text{Nyq}} = 1/T_{\text{Nyq}}$ the Nyquist rate of $x(t)$. Only N_{Sig} and B, or at least an upper bound for each, are assumed to be known. The carrier frequencies and modulations of $s_i(t)$ are unknown. Denote the frequency support of $x(t)$ by $S = S(x(t))$ and by $\kappa = 2N_{\text{Sig}}$ its sparsity. The signal is received by a CR network composed of N_{rec} sensors.

B. Network Model

The CR network is modeled by an undirected, connected graph $G = (V; E)$ where V is the set of receivers and E is the set of communication links. The existence of edge $(i, j) \in E$ means that the ith and jth CRs can exchange messages on a control channel. Denote the neighbor set of the ith CR by $N(i)$, i.e. $N(i) = \{j|(i, j) \in E\}$ and its cardinality by d_i, i.e. $d_i = |N(i)|$. The communication is assumed to be reliable, namely no messages are lost.

C. Faded Received Signal

We consider two effects of the transmission channels: Rayleigh fading, or small-scale fading, and log-normal shadowing, or large-scale fading [20], [3], [21]. Denote by $r_{ij}(t)$ the received signal corresponding to the ith transmission, $1 \leq i \leq N_{\text{sig}}$, received at the jth CR, $1 \leq j \leq N_{\text{rec}}$. The received signal is generally described in terms of the transmitted signal $s_i(t)$ convolved with the impulse response of the channel $h_{ij}(t)$, namely

$$r_{ij}(t) = s_i(t) * h_{ij}(t), \tag{3}$$

where $*$ denotes convolution.

1) Rayleigh fading: For most practical channels, the free-space propagation model, which only accounts for path loss, is inedequate to describe the channel. A signal can travel from transmitter to receiver over multiple reflective paths, which is traditionally modeled as Rayleigh fading, namely the envelope of the channel responses $h_{ij}(t)$ follows the Rayleigh distribution, given by

$$p_h(r) = \begin{cases} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} & r \geq 0 \\ 0 & otherwise, \end{cases} \tag{4}$$

where r is the enveloppe amplitude of the received signal, and $2\sigma^2$ its mean power [21].

2) Log-normal shadowing: Large-scale fading represents the average signal power attenuation or path loss due to motion over large areas. This phenomenon is affected by prominent terrain contours between the transmitter and receiver. Empirical measurements suggest that this type of fading, or shadowing, follows a normal distribution in dB units [22], or alternatively, the linear channel gain may be modeled as a log-normal random variable [3]. Therefore, the path loss (PL) measured in dB is expressed as

$$PL = PL_0 + 10\gamma \log \frac{d}{d_0} + X_\sigma. \tag{5}$$

Here, the reference distance d_0 corresponds to a point located in the far field of the antenna (typically 1 km for large cells). The path loss to the reference point PL_0 is usually found through field measurements or calculated using free-space path loss. The value of the path loss exponent γ depends on the frequency, antenna heights, and propagation environment. Finally, X_σ denotes a Gaussian random variable (in dB) with variance σ^2 determined heuristically as well [21].

The shadowed received signal is thus given by

$$r_{ij}(t) = 10^{-PL_{ij}/20} \cdot s_i(t), \tag{6}$$

where PL_{ij} denotes the path loss between the ith transmitter and the jth receiver. Here, $h_{ij}(t) = 10^{-PL_{ij}/20}$ is a constant.

D. Problem Formulation

A network of N_{rec} CRs receives the N_{sig} transmissions, such that the received signal at the jth CR is given by

$$x^{(j)}(t) = \sum_{i=1}^{N_{\text{sig}}} r_{ij}(t). \tag{7}$$

Obviously, the support of $x^{(j)}(t)$ is included in the support of $x(t)$, namely $S(x^{(j)}(t)) \subseteq S$. Since the transmissions are affected differently by fading and shadowing effects from each transmitter to each CR, we assume that $\bigcup \left(S(x^{(j)}(t))\right) = S$.

Our goal is therefore to assess the support of the transmitted signal $x(t)$ from sub-Nyquist samples of the received $x^{(j)}(t), 1 \leq j \leq N_{\text{rec}}$. In order to determine the support of $x(t)$, we exploit the joint sparsity shared by $x^{(j)}(t), 1 \leq j \leq N_{\text{rec}}$. Specifically, we jointly recover the common support of $x^{(j)}(t)$ from their sub-Nyquist samples, in a distributed manner.

III. Individual Sub-Nyquist Sampling

In this section, we briefly describe the sub-Nyquist sampling schemes performed at each CR on the corresponding received signal $x^{(j)}(t)$. We consider two different approaches: multicoset sampling [23] and the MWC [17] which were previously proposed for sparse multiband signals. Due to lack of space, we only briefly describe the MWC sampling scheme. The reader is refered to [23] for more details on multicoset sampling. However, since both schemes lead to identical expressions of the signal spectrum in terms of the samples, the support reconstruction stage presented in Section IV can be applied to either of the samples. For convenience, we drop the index j in this section.

A. MWC sampling

The MWC [17] is composed of M parallel channels. In each channel, an analog mixing front-end, where $x(t)$ is multiplied by a mixing function $p_i(t)$, aliases the spectrum, such that each band appears in baseband. The mixing functions $p_i(t)$ are required to be periodic with period T_p such that $f_p = 1/T_p \geq B$. The function $p_i(t)$ has a Fourier expansion

$$p_i(t) = \sum_{l=-\infty}^{\infty} c_{il} e^{j\frac{2\pi}{T_p}lt}. \tag{8}$$

In each channel, the signal goes through a lowpass filter with cut-off frequency $f_s/2$ and is sampled at the rate $f_s \geq f_p$. For the sake of simplicity, we choose $f_s = f_p$. Repeating the calculations in [17], we derive the relation between the known DTFTs of the samples $y_i[n]$ and the unknown $X(f)$

$$\mathbf{z}(f) = \mathbf{A}\mathbf{x}(f), \qquad f \in \mathcal{F}_s, \tag{9}$$

where $\mathbf{z}(f)$ is a vector of length N with ith element $z_i(f) = Y_i(e^{j2\pi f T_s})$. The unknown vector $\mathbf{x}(f)$ is given by

$$\mathbf{x}_k(f) = X\left(f + K_k f_s\right), \quad 1 \leq k \leq N, \tag{10}$$

where $K_k = k - \frac{N+1}{2}, 1 \leq k \leq N$ for odd N and $K_k = k - \frac{N+2}{2}, 1 \leq k \leq N$ for even N. The $M \times N$ matrix $\mathbf{A}$ contains the coefficients c_{il} such that $\mathbf{A}_{il} = c_{i,-l} = c^*_{il}$. The overall sampling rate is

$$f_{tot} = M f_s = \frac{M}{N} f_{\text{Nyq}}. \tag{11}$$

B. Continuous to Finite (CTF)

The set of equations (9) consist of an infinite number of linear systems since f is a continuous variable. Such systems are known as infinite measurement vectors (IMV) in the compressed sensing (CS) literature. We use the support recovery paradigm from [23] that produces a finite system of equations, called multiple measurement vectors (MMV) from an infinite number of linear systems. This reduction is performed by what is referred to as the continuous to finite (CTF) block.

From (9), we have

$$\mathbf{Q} = \mathbf{A}\mathbf{Z}\mathbf{A}^H \tag{12}$$

where $\mathbf{Q} = \int_{f \in \mathcal{F}_s} \mathbf{z}(f)\mathbf{z}^H(f)\mathrm{d}f$ is a $M \times M$ matrix and $\mathbf{Z} = \int_{f \in \mathcal{F}_s} \mathbf{x}(f)\mathbf{x}^H(f)\mathrm{d}f$ is a $N \times N$ matrix. We then construct a frame $\mathbf{V}$ such that $\mathbf{Q} = \mathbf{V}\mathbf{V}^H$. Clearly, there are many possible ways to select $\mathbf{V}$. We construct it by performing an eigendecomposition of $\mathbf{Q}$ and choosing $\mathbf{V}$ as the matrix of eigenvectors corresponding to the non zero eigenvalues. We can then define the following linear system

$$\mathbf{V} = \mathbf{A}\mathbf{U}. \tag{13}$$

From [23] (Propositions 2-3), the support of the unique sparsest solution of (13) is the same as the support of the original set of equations (9). In order to find the joint support S, we exploit both the simultaneous sparsity between the columns $\mathbf{U}^{(j)}$ for each CR and the joint sparsity between the matrices $\mathbf{U}^{(j)}$ between all CRs.

In the worst case, we require $M \geq 2\kappa$, leading to a minimal sampling rate of $2\kappa B$ for each CR [23].

IV. Joint Support Reconstruction

In this section, we consider joint support recovery from the observation matrices $\mathbf{V}^{(j)}, 1 \leq j \leq N_{\text{rec}}$. We present two distributed algorithms, DOSGA and RDSIHT. While the latter is our main contribution, the former is derived as a simple alternative for comparison purposes, extending the OSGA, presented in [19].

A. DOSGA

We first adapt the OSGA to our distributed settings. The goal is for each CR to learn the $N \times 1$ vector $\hat{\mathbf{w}}$ that approximates the average of $\mathbf{w}^{(j)}, 1 \leq j \leq N_{\text{rec}}$, i.e. $\hat{\mathbf{w}} = \frac{1}{N_{\text{rec}}} \sum_{j=1}^{N_{\text{rec}}} \mathbf{w}^{(j)}$. Here, the nth row of $\mathbf{w}^{(j)}$, namely $\mathbf{w}^{(j)}_n = \left\| \left(\left(\mathbf{A}^{(j)}\right)^H \mathbf{V}^{(j)} \right)^T_n \right\|^2_2$ computes the ℓ_2-norm of the projection of the observation matrix $\mathbf{V}^{(j)}$ onto the nth column of the measurement matrix $\mathbf{A}^{(j)}$, where $(\mathbf{C})^T_n$ denotes the nth row of the matrix $\mathbf{C}$. Finding this average is a standard distributed average consensus problem, also referred to as distributed averaging or distributed consensus. We use a randomized gossip algorithm [24] for this purpose.

At each iteration, a CR, say with index i, is chosen uniformly at random (i.e. it wakes up with some probability). It then contacts a random neighbor j, chosen with some probability P_{ij}, according to the Metropolis-Hastings scheme for random transition probabilities,

$$P_{ij} = \begin{cases} min\{\frac{1}{d_i}, \frac{1}{d_j}\} & (i,j) \in E \\ \sum_{(i,k) \in E} max\{0, \frac{1}{d_i} - \frac{1}{d_k}\} & i = j \\ 0 & otherwise. \end{cases} \tag{14}$$

and computes the average between $\mathbf{w}^{(i)}$ and $\mathbf{w}^{(j)}$. Both CRs update their correlation vector and corresponding support accordingly. The resulting DOSGA is shown in Algorithm 1. Here, $\mathrm{supp}(\mathcal{T}(\mathbf{w},\kappa))$ denotes the κ largest elements of $\mathbf{w}$.

Algorithm 1 DOSGA

Input: observation matrices $\mathbf{V}^{(j)}$, measurement matrices $\mathbf{A}^{(j)}$, random transition probabilities matrix $\mathbf{P}$

Output: index set S containing the joint support of $\mathbf{U}^{(j)}$

Initialization: $\mathbf{w}_n^{(j)} = \left\| \left(\left(\mathbf{A}^{(j)}\right)^H \mathbf{V}^{(j)} \right)_n^T \right\|_2^2, 1 \le n \le N$, $S^{(j)} \leftarrow \mathrm{supp}(\mathcal{T}(\mathbf{w}^{(j)},\kappa))$, for $1 \le j \le N_{\mathrm{Nrec}}$, $i \leftarrow 1$

while halting criterion false **do**

  Select neighbor j with probability $\mathbf{P}_{ij}$

  $\mathbf{c} = \frac{\mathbf{w}^{(i)}+\mathbf{w}^{(j)}}{2}$

  $\mathbf{w}^{(i)} = \mathbf{c}$, $\mathbf{w}^{(j)} = \mathbf{c}$

  $S^{(i)} \leftarrow \mathrm{supp}(\mathcal{T}(\mathbf{w}^{(i)},\kappa))$, $S^{(j)} \leftarrow \mathrm{supp}(\mathcal{T}(\mathbf{w}^{(j)},\kappa))$

end while

$S \leftarrow S^{(i)}$

DOSGA converges to the same solution as centralized OSGA.

B. RDSIHT

Next, we turn to the RDSIHT algorithm, which adapts the centralized BSIHT algorithm [18] (Algorithm 2) to the distributed scenario. BSIHT is itself an extension of SIHT [25] to the block sparse case. In the collaborative centralized problem, where a fusion center exists, each receiver contributes a jointly sparse matrix $\mathbf{U}^{(j)}$. Estimates of the columns of the sparse matrix $\mathbf{U}^{(j)}$ are computed separately. The indices of the common support are then selected by averaging over the estimates of the jointly sparse columns of all receivers. Once the support is selected, the updated calculations are performed separately for each column of the sparse matrix and for each receiver. An adaptive step size is used to improve the performance with regard to a fixed scaling factor [25].

Our distributed approach was inspired by the randomized incremental subgradient method proposed in [26] and recent work on a stochastic version of IHT in [27]. A vector $\mathbf{w}$ of size N is shared in the network through random walk. The indices of its κ largest values correspond to the current estimated support. When the ith CR receives $\mathbf{w}$, it locally updates it by performing a gradient step using its own objective function that is then added to $\mathbf{w}$. Next, it selects a neighbor j to send the vector to with probability P_{ij} (14). The joint sparsity accross the CRs is exploited by sharing one common vector $\mathbf{w}$ by the network. The resulting RDSIHT is shown in Algorithm 2. Here, $\hat{\mathbf{U}}_{|S}$ is the estimated matrix $\mathbf{U}$ reduced to the support set S and $(\cdot)^H$ denotes the Hermitian operation.

The communication load of DOSGA is twice that of RDSIHT for the same number of iterations, since in DOSGA, two vectors are sent per iteration, whereas in RDSIHT, only one vector is sent per iteration.

V. Simulation Results

In this section, we compare the performance of our two algorithms, DOSGA and RDSIHT, along with the centralized BSIHT [18], in order to test the convergence of the distributed RDSIHT to its centralized version.

Algorithm 2 RDSIHT

Input: observation matrices $\mathbf{V}^{(j)}$, measurement matrices $\mathbf{A}^{(j)}$, random transition probabilities matrix $\mathbf{P}$

Output: index set S containing the joint support of $\mathbf{U}^{(j)}$

Initialization: $\mathbf{w} = 0$, $\hat{\mathbf{U}}^{(j)} = 0$, $S^{(j)} \leftarrow \{1,...,\kappa\}$, $\mu^{(j)} = \frac{1}{M}$, $\mathbf{Z}^{(j)} = 0$, $\mathbf{c}^{(j)} = 0$, for $1 \le j \le N_{\mathrm{Nrec}}$, $i \leftarrow 1$

while halting criterion false **do**

  $\mathbf{Z}^{(i)} = \hat{\mathbf{U}}^{(i)} + \mu^{(i)}\left(\mathbf{A}^{(i)}\right)^H \left(\mathbf{V}^{(i)} - \mathbf{A}^{(i)}\hat{\mathbf{U}}^{(i)}\right)$

  $\mathbf{w}_n = \mathbf{w}_n + \left\| \left(\mathbf{Z}^{(i)}\right)_n^T \right\|_2^2, 1 \le n \le N$

  $S^i \leftarrow \mathrm{supp}(\mathcal{T}(\mathbf{w},\kappa))$

  $\hat{\mathbf{U}}^{(i)} = \mathbf{Z}^{(i)}{}_{|_{S^{(i)}}}$

  $\mu^{(i)} = \dfrac{\left\| \left[\left(\mathbf{A}^{(i)}\right)^H \left(\mathbf{V} - \mathbf{A}^{(i)}\hat{\mathbf{U}}^{(i)}\right) \right]_{|_{S^{(i)}}} \right\|}{\left\| \mathbf{A}^{(i)}{}_{|_{S^{(i)}}} \left[\left(\mathbf{A}^{(i)}\right)^H \left(\mathbf{V} - \mathbf{A}^{(i)}\hat{\mathbf{U}}^{(i)}\right) \right]_{|_{S^{(i)}}} \right\|}$

  Select neighbor j with probability $\mathbf{P}_{ij}$

  $i \leftarrow j$

end while

$S \leftarrow S^{(i)}$

In the simulations, we consider signals $x(t)$ with Nyquist rate $f_{\mathrm{Nyq}} = 6.1$GHz composed of $N_{\mathrm{sig}} = 3$ QPSK modulated transmissions with arbitrary carriers and single-sided bandwidth $B = 20$MHz. The transmissions are passed through Rayleigh channels with maximum shifting $2\sigma^2 = 5$MHz. Besides, we apply log-normal shadowing with the following parameters: reference distance $d_0 = 0.01$, path loss to the reference point $PL_0 = 0$, γ and X_σ are chosen arbitrarily from the sets of values $\{2.6, 2.4, 0, 3\}$, $\{14.1, 9.6, 0, 7\}$ respectively. These are common values describing different obstacles and propagation effects [28].

In each experiment, a set of N_{rec} 2D points, representing the coordinates of the CRs, are generated uniformly at random over the unit square. We consider a geometric graph [29] where a pair of nodes is connected if the euclidean distance between them is less than $d_{\mathrm{neigh}} = 0.5$. In addition, the distances between each CR and each transmitter are drawn uniformly at random between 0 and 1. For the sampling stage at each CR, we consider $N = 256$ spectral bands and $M = 15$ analog channels, each sampling at $f_s = 24$MHz and with $N_s = 40$ samples per channel. The overall sampling rate of each receiver is thus 360MHz, which is a little below 6% of the Nyquist rate and 3 times the Landau rate.

In all of the three algorithms, the sparsity is assumed to be known. DOSGA and RDSIHT use a maximum number of iterations of 500 as halting criterion, whereas BSIHT runs for up to 10 iterations. The number of iterations of the distributed algorithm is large enough for all the CRs to converge to the same support. A success is declared whenever the joint recovered and original supports are exactly identical. Each experiment is repeated over 500 realisations.

We show the influence of several practical parameters on the performance of our recovery algorithms. In the first experiment, we illustrate the impact of signal-to-noise ratio (SNR) on the detection performance. Here, we consider $N_{\mathrm{rec}} = 20$.

Figure 1 shows the support recovery success rate of the three algorithms for different values of SNR.

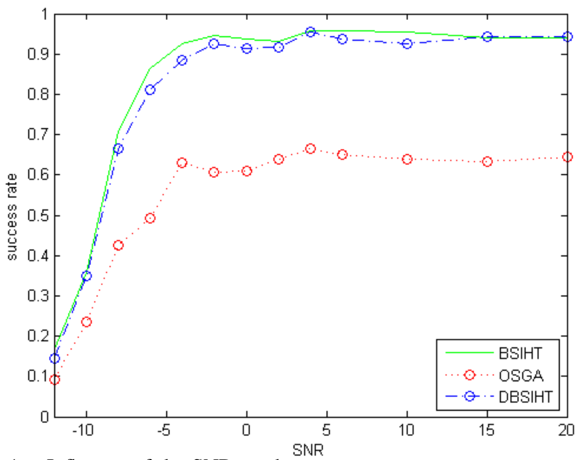

Fig. 1. Influence of the SNR on the success rate.

In the second experiment, we vary the number of receivers N_{rec}. We consider the same sampling parameters as in the previous experiment and set the SNR to be 10dB. Figure 2 shows the support recovery success rate for different values of the number of receivers.

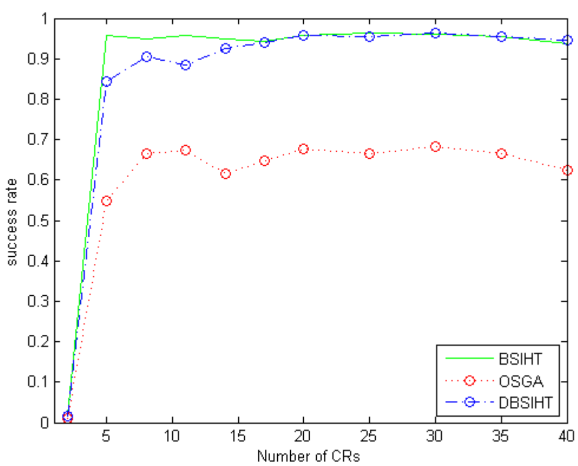

Fig. 2. Influence of the number of receivers on the success rate.

We observe that RDSIHT outperforms DOSGA for different parameters combination. In addition, RDSIHT converges in performance to its centralized version, BSIHT.

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